

APPM 1345

Exam 3

Spring 2024

Name		
Instructor	Richard McNamara	Section 150

1. (23 pts) Parts (a) and (b) are unrelated.

(a) Find the inverse function of $f(x) = \frac{\ln(2x)}{1 + \ln(2x)}$ for $x > \frac{1}{2}$.

Express your answer in the form $f^{-1}(x)$. (You do not have to identify the inverse function's domain.)

(b) Consider the function $g(x) = 2x - \cos x$.

i. Explain why g is invertible, based on its derivative.

ii. Find an equation of the line that is tangent to the curve $y = g^{-1}(x)$ at the point $(4 - 1; 2)$.

Hint: Do not attempt to identify the function $g^{-1}(x)$.

2. (25 pts) Parts (a) and (b) are unrelated.

- (a) If a substance undergoing exponential decay has a half-life of 50 years, how many years would it take for a sample of that substance to decay to 1 percent of its original amount?

- (b) Consider the function $p(t) = p_0 e^{kt}$, which represents an exponential growth model for a population, where the constant p_0 represents the initial population size and the constant k represents the population's relative growth rate. Suppose $p(10) = 2$ and $p(50) = 6$.
- i. Find the value of k .
 - ii. Find the value of p_0 .

3. (26 pts) Evaluate the following derivatives using properties of logarithms and/or logarithmic differentiation. Do **not** fully simplify your answers, although they must be expressed as functions of x .

(a) $\frac{d}{dx} \ln \frac{(10 \cos^2 x)^{10} (x^4 + 6)^{10}}{e^{x \sin x}}$ #

$$(b) \frac{d}{dx} e^x + e^{x^x}$$

4. (26 pts) Evaluate the following integrals.

(a) $\int_1^2 \frac{2^x}{9 \cdot 2^x} dx$

(b) $\int \frac{x}{x-1} dx$

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.